Numerical Modeling of Inclined Negatively Buoyant Jets

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Outline

• Introduction
• Objectives
• Definitions
• Numerical Details
• Results
• Conclusions
• Future Work
• References

Hollywood sewage outfall

Numerical Modeling of Inclined Negatively Buoyant Jets
Introduction

Effluent (USEPA Definition): Wastewater, treated or untreated, that flows out of a treatment plant, sewer, or industrial outfall. Generally refers to wastes discharged into surface waters.

Effluent Sources

- Desalination Plants (arid and semi-arid countries)
- Nuclear Power Plants
- Municipal Effluents

Al Ghubrah desalination plant (biggest in Oman). (photo by Hamdi Al-Barwani)
From: http://www.cbc.ca
St. Lawrence River. From: EC

Numerical Modeling of Inclined Negatively Buoyant Jets
Introduction (Cont’d)

Effluent Discharges into the water body:

1. Surface Discharges

2. Submerged Discharges

From: www.sciencephoto.com
Introduction (Cont’d)

Submerged Discharges

a. Negatively Buoyant Jets  b. Positively Buoyant Jets

Numerical Modeling of Inclined Negatively Buoyant Jets
Objectives

• Evaluating the performance of numerical model
• Finding the appropriate numerical model/solver
• Implementations in the base-code
• Evaluation of various turbulence models
• Finding the stable numerical schemes
Definitions

• Dilution

\[ S = \frac{C_0 - C_a}{C - C_a} \]

- \( C_0 \): Concentration at Source
- \( C_a \): Ambient Concentration
- \( C \): Concentration at Mesh Grid
Definitions (Cont’d)

Numerical Modeling of Inclined Negatively Buoyant Jets
Numerical Details

Governing Equations

Cont. \( \frac{\partial u_j}{\partial x_j} = 0 \)

Mom. \( \frac{\partial u_i}{\partial t} + \frac{\partial}{\partial x_j} (u_j u_i) = - \frac{1}{\rho} \frac{\partial P}{\partial x_i} + \frac{\partial}{\partial x_j} (\nu_{eff} \frac{\partial u_i}{\partial x_j}) - g_i \frac{\Delta \rho}{\rho} \)

Temp. \( \frac{\partial T}{\partial t} + \frac{\partial T u_j}{\partial x_j} = k_{eff} \frac{\partial}{\partial x_k} (\frac{\partial T}{\partial x_k}) \)

Con. \( \frac{\partial C}{\partial t} + \frac{\partial C u_j}{\partial x_j} = D \frac{\partial}{\partial x_k} (\frac{\partial C}{\partial x_k}) \)

Heat transfer coefficient \( k_{eff} = \frac{\nu_t}{Pr_t} + \frac{\nu_0}{Pr_0} \)

Effective kinematic viscosity \( \nu_{eff} = \nu_0 + \nu_t \)
The OpenFOAM (OPEN Field Operation And Manipulation) CFD Toolbox is a free, open source CFD software package produced by OpenCFD Ltd (2011).

Advantages:

• Open Source

• Finite Volume Method

• Working on LINUX OS

• Variety of Utilities and Applications
Numerical Details (Cont’d)

Solver

Solver: mypisoFoam
   A transient solver for incompressible flow

Transport Eqns. for U, S and T are solved implicitly

P is solved explicitly in PISO algorithm

Density varies with S and T (Millero and Poisson, 1981)

\[ \rho = \rho_i + AS + BS^{3/2} + CS \]
\[ A = 8.24493 \times 10^{-1} - 4.0899 \times 10^{-3} T + 7.6438 \times 10^{-5} T^2 - 8.2467 \times 10^{-7} T^3 + 5.3875 \times 10^{-9} T^4 \]
\[ B = -5.72466 \times 10^{-3} + 1.0227 \times 10^{-4} T - 1.6546 \times 10^{-6} T^2 \]
\[ C = 4.8314 \times 10^{-4} \]
\[ \rho_i = 999.842594 + 6.793952 \times 10^{-2} T - 9.095290 \times 10^{-3} T^2 + 1.001685 \times 10^{-4} T^3 - 1.120083 \times 10^{-6} T^4 + 6.536336 \times 10^{-9} T^5 \]
Numerical Details (Cont’d)

Simulation process priority in OF

- Geometry
- Mesh
- Solver
- Postprocessing

Numerical Modeling of Inclined Negatively Buoyant Jets
Numerical Details (Cont’d)

Turbulence Modeling

Seven RANS Turbulence Models:

- Four LEVMs
  - Standard k-ε
  - RNG k-ε
  - Realizable k-ε
  - SST k-ω

- Two RSMs
  - Launder-Gibson
  - LRR

- One NLEVM: nonlinear k-ε

Buoyant wall jet study

Inclined dense jet study

Numerical Modeling of Inclined Negatively Buoyant Jets
Results

45° inclined dense jet, LRR turbulence model
# Numerical test cases

<table>
<thead>
<tr>
<th>Case</th>
<th>Inclined Angle $\Theta$</th>
<th>Initial Inlet Height $y_0$ (mm)</th>
<th>D (mm)</th>
<th>$U_0$ (m/s)</th>
<th>$\Delta\rho/\rho_0$ (%)</th>
<th>$F_d$</th>
<th>$L_m$ (mm)</th>
<th>$y_0/L_m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>30.00</td>
<td>11.17</td>
<td>6.50</td>
<td>1.00</td>
<td>1.98</td>
<td>28.10</td>
<td>172.00</td>
<td>998.20</td>
</tr>
<tr>
<td>2</td>
<td>45.00</td>
<td>12.92</td>
<td>6.00</td>
<td>1.17</td>
<td>1.98</td>
<td>34.30</td>
<td>193.00</td>
<td>999.97</td>
</tr>
</tbody>
</table>
Results (Cont’d)

Normalized terminal rise height as a function of initial discharge angle

Numerical Modeling of Inclined Negatively Buoyant Jets
Minimum dilution at the return point as a function of initial discharge angle

- Exp. Roberts and Toms (1987)
- Exp. Roberts et al. (1997)
- Exp. Shao and Law (2010), Far from boundary
- Exp. Shao and Law (2010), Near to boundary
- Num. RNGkEpsilon
- Num. realizableKE
- Num. NonlinearKE
- Num. LaunderGibson
- Num. LRR

Numerical Modeling of Inclined Negatively Buoyant Jets
## Comparison of numerical and experimental coefficients for 45° inclined jets

<table>
<thead>
<tr>
<th></th>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Terminal rise height</td>
<td>( C_1 = \frac{y_0}{D} Fr )</td>
<td>1.44</td>
<td>1.54</td>
<td>1.75</td>
<td>1.46</td>
<td>1.52</td>
<td>Avg=1.62</td>
</tr>
<tr>
<td>Horizontal location of return point</td>
<td>( C_2 = \frac{x_r}{D} Fr )</td>
<td>2.80</td>
<td>3.32</td>
<td>3.79</td>
<td>2.96</td>
<td>3.18</td>
<td>Avg=3.08</td>
</tr>
<tr>
<td>Return point dilution</td>
<td>( C_3 = \frac{S_r}{Fr} )</td>
<td>0.86</td>
<td>1.20</td>
<td>0.82</td>
<td>1.04</td>
<td>1.11</td>
<td>Avg=1.48</td>
</tr>
<tr>
<td>Vertical location of centerline peak</td>
<td>( C_4 = \frac{y_m}{D} Fr )</td>
<td>1.05</td>
<td>1.19</td>
<td>1.39</td>
<td>1.10</td>
<td>1.13</td>
<td>Avg=1.48</td>
</tr>
<tr>
<td>Horizontal location of centerline peak</td>
<td>( C_5 = \frac{x_m}{D} Fr )</td>
<td>1.57</td>
<td>1.75</td>
<td>2.04</td>
<td>1.65</td>
<td>1.72</td>
<td>Avg=1.80</td>
</tr>
<tr>
<td>Centerline peak dilution</td>
<td>( C_6 = \frac{S_m}{Fr} )</td>
<td>0.39</td>
<td>0.45</td>
<td>0.37</td>
<td>0.42</td>
<td>0.44</td>
<td>Avg=0.44</td>
</tr>
</tbody>
</table>

### Numerical Modeling of Inclined Negatively Buoyant Jets
Results (Cont’d)
Inclined dense jet

Normalized concentration profiles at various downstream cross-sections for a 30° jet

Numerical Modeling of Inclined Negatively Buoyant Jets
Results (Cont’d)
Inclined dense jet

Comparison of concentration spread width along the trajectory. Upper $b_c$
Conclusions

- Numerical results of selected turbulence models show good agreement for the velocity and concentration fields between both experimental and numerical studies.

- Realizable k-ε and LRR turbulence models performed best amongst the seven models investigated.

- Geometrical characteristics of inclined dense jets have been predicted fairly well.

- Cross-sectional U & C profiles follow the Gaussian pattern better in outer-half of the jet as well as closer area to source than the inner-half.
Future Work

• Improved mesh grid system: unstructured, non-conformal, etc.

• Improved turbulence models

• More advanced numerical schemes

• Ambient water characteristics: cross-flow, stratification, wave, etc.
References


Thank you!
Thank you!

TO MAKE PROGRESS, I EXPECT ALL MY STUDENTS TO BE AT WORK FROM 9 TO 5.

YOU MEAN "AM" OR "PM"?

BOTH.

MATHEMATICALLY, THAT STILL LEAVES YOU 8 HOURS TO SLEEP.

MATHEMATICALLY, I WOULDN'T HAVE A LIFE.

MATH IS HARD.

Numerical Modeling of Turbulent Wall Jets in Stationary Ambient Water
Definitions

General

**Dimensional Analysis**

\[ F_d = \frac{U_0}{\sqrt{g' D}} \]

- **Densimetric Froude #**

\[ g' = g \frac{\rho_a - \rho_0}{\rho_0} \]

- **Momentum Length Scale**

\[ L_M = \frac{M_0^{3/4}}{B_0^{1/2}} \]

- **Source Length Scale**

\[ L_Q = \frac{Q_0}{M_0^{1/2}} \]
Numerical Details

Numerical Modeling Procedure

- Mathematical Model (PDEs, BC)
- Discretization Method (FDM, FVM, FEM)
- Finite Approximation (Numerical Schemes)
- Solution Method
- Convergence Criteria (Stopping Condition)
Numerical Details (Cont’d)

FVM

OpenFOAM

Numerical Modeling of Thermal/Saline Discharges in Coastal Waters
Numerical Schemes: 1\textsuperscript{st} and 2\textsuperscript{nd} order schemes

\[
\text{div}(\phi, S) \quad \underbrace{\text{Gauss}}_{\text{Gaussian Integration}} \quad \underbrace{\text{upwind} \;}_{\text{Interpolation Scheme}}
\]

Numerical Solution

```plaintext
U
{
    solver PBiCG;
    preconditioner DILU;
    tolerance 1e-05;
    relTol 0;
}
```
Numerical Details (Cont’d)
Turbulence Modeling

Algebraic Models: An algebraic Eqn. for turbulent viscosity

1 Eqn. Models: A transport Eqn. is solved (for turbulent kinetic energy)

2 Eqn. Models: Two transport Eqn. is solved (e.g. for k & ε)

Reynolds Stress: $\rho \overline{w_i u_j} = -(\mu_t)(\overline{U_{i,j}} + \overline{U_{j,i}})$

RSM: A transport Eqn. for Reynolds stress tensor
Results (Cont’d)
Buoyant wall jet

Centerline trajectory. Fr # about 20
Results (Cont’d)
Buoyant wall jet

Comparison of the maximum velocity decay
Results (Cont’d)

Buoyant wall jet

Spanwise w-velocity profiles at $y=y_m$ for case # 3
Results (Cont’d)
Buoyant wall jet

Streamwise temperature profiles for case # 3
Results (Cont’d)
Buoyant wall jet

Spanwise temperature profiles for case # 3

Numerical Modeling of Thermal/Saline Discharges in Coastal Waters
Results (Cont’d)
Buoyant wall jet

Comparison of the maximum temperature decay
Results (Cont’d)
Inclined dense jet

Normalized centerline trajectory for 30° inclined dense jet

Normalized centerline trajectory for 45° inclined dense jet
Results (Cont’d)

Inclined dense jet

Normalized vertical location of centerline peak as a function of initial discharge angle

- Exp. Cipolina et al. (2005)
- Exp. LA data (Kikkert, 2006)
- Exp. LIF data (Kikkert, 2006)
- Exp. Shao and Law (2010)
- Num. RNGkEpsilon
- Num. realizableKE
- Num. NonlinearKE
- Num. LaunderGibson
- Num. LRR

Theory Ferrari and Querzoli (2004)
Upper and Lower limit for Ferrari and Querzoli (2004)
Theory (Kikkert, 2006)
Results (Cont’d)
Inclined dense jet

Normalized horizontal location of return point as a function of initial discharge angle
Results (Cont’d)
Inclined dense jet

Normalized variation of dilution along the inlet height level for a 35° jet
Inclined dense jet

Comparison of normalized centerline max velocity decay for a 30° jet

Comparison of normalized centerline max velocity decay for a 45° jet
Results (Cont’d)
Inclined dense jet

Cross-sectional C distribution at various downstream locations
Results (Cont’d)
Inclined dense jet

Normalized velocity profiles at various downstream cross-sections for a 30° jet